and the third in 1735) in studying the *Euclides Vindicatus*. He has also appended two pages of notes, a subject index, and an index of proper names. The whole has been issued by the Open Court Publishing Company in a volume of attractive appearance.

This enterprising company and Dr. Halsted have placed mathematicians much in their debt by making the masterpieces of Saccheri,[1] Bolyai, and Lobatchevsky so readily accessible to American and English readers.

R. C. ARCHIBALD.

The Elementary Differential Geometry of Plane Curves. By R. H. FOWLER. (Cambridge Tracts in Mathematics and Mathematical Physics, No. 20.) Cambridge, at the University Press, 1920. 7 + 105 pages. Price 6 shillings.

Preface: "This tract is intended to present a precise account of the elementary differential properties of plane curves. The matter contained is in no sense new, but a suitable connected treatment in the English language has not been available.

"As a result, a number of interesting misconceptions are current in English text books. It is sufficient to mention two somewhat striking examples. (*a*) According to the ordinary definition of an envelope, as the locus of the limits of points of intersection of neighbouring curves, a curve is not the envelope of its circles of curvature, for neighbouring circles of curvature do not intersect. (*b*) The definitions of an asymptote—(1) a straight line, the distance from which of a point on the curve tends to zero as the point tends to infinity; (2) the limit of a tangent to the curve, whose point of contact tends to infinity—are not equivalent. The curve may have an asymptote according to the former definition and the tangent may exist at every point, but have no limit as its point of contact tends to infinity.

"The subjects dealt with, and the general method of treatment, are similar to those of the usual chapters on geometry in any *Cours d'Analyse*, except that in general plane curves alone are considered. At the same time extensions to three dimensions are made in a somewhat arbitrary selection of places, where the extension is immediate, and forms a natural commentary on the two dimensional work, or presents special points of interest (Frenet's formulæ). To make such extensions systematically would make the tract too long. The subject matter being wholly classical, no attempt has been made to give full references to sources of information; the reader however is referred at most stages to the analogous treatment of the subject in the *Cours* or *Traité d'analyse* of de la Vallée Poussin, Goursat, Jordan or Picard, works to which the author is much indebted.

"In general the functions, which define the curves under consideration, are (as usual) assumed to have as many continuous differential coefficients as may be mentioned. In places, however, more particularly at the beginning, this rule is deliberately departed from, and the greatest generality is sought for in the enunciation of any theorem. The determination of the *necessary and sufficient* conditions for the truth of any theorem is then the primary consideration. In the proofs of the elementary theorems, where this procedure is adopted, it is believed that this treatment will be found little more laborious than any rigorous treatment, and that it provides a connecting link between Analysis and more complicated geometrical theorems, in which insistence on the precise necessary conditions becomes tedious and out of place, and suitable sufficient conditions can always be tacitly assumed. At an earlier stage the more precise formulation of conditions may be regarded as (1) an important grounding for the student of Geometry, and (2) useful practice for the student of Analysis.

"The introductory chapter is a collection of somewhat disconnected theorems which are

[1] For other discussions of Saccheri's *Euclides Vindicatus* the curious reader may turn to Cantor, *Vorlesungen über Geschichte der Mathematik*, Vol. 3, 2te Aufl., 1901; to P. Mansion, "Analyse des recherches du P. Saccheri, S. J., sur le postulatum d'Euclide," *Annales de la Société scientifique de Bruxelles*, 1889–1890, Vol. 14, 2d part; reprinted in a supplement to *Mathesis*, January, 1891; to G. Veronese, *Grundzüge der Geometrie*, Leipzig, 1894, pp. 636–639; and to H. S. Carslaw's English translation of Bonola's *Non-Euclidean Geometry* (Open Court, 1912).

It seems curious that there is no reference to Saccheri in *The Catholic Encyclopedia*.

required for reference. The reader can omit it, and refer to it as it becomes necessary for the understanding of later chapters."

Contents—Chapter I: Introduction, 1–8; II: The elementary properties of tangents and normals, 8–23; III: The curvature of plane curves, 24–44; IV: The theory of contact, 45–58; V: The theory of envelopes, 58–79; VI: Singular points of plane curves, 80–89; VII: Asymptotes of plane curves, 89–103.

NOTES.

The second edition of W. A. Robertson and F. A. Ross's *Actuarial Theory* (Edinburgh, Oliver and Boyd, 23 + 431 pages) appeared in the early summer of 1920. The changes in the first edition are mainly in the correction of errors.

The Oxford University Press announces the following books: *Roger Bacon and the State of Science in the Fourteenth Century* by R. Steele; *Archimedes' Principle of the Balance and some criticisms upon it* by J. M. Child; *A History of Greek Mathematics*, 2 volumes, by Sir Thomas Heath—The Cambridge University Press has published a third edition of E. T. Whittaker and G. N. Watson's *A Course of Modern Analysis* (8 + 608 pages; price 40 shillings). It contains about 230 pages more than the first edition by Whittaker alone.

The second part of the sixth volume of the *Encyklopädie der Mathematischen Wissenschaften* is devoted to Astronomy. Between 1905 and 1915 six Hefte were published; the last of these included Professor E. W. BROWN'S "Theorie des Erdmondes," translated by A. v. Brunn. The first thirty-five pages of Heft 7, published in 1920, contain "Die Satelliten" by Professor KURT LAVES, of the University of Chicago. This section was "abgeschlossen im Sommer 1916." The rest of the Heft (pages 843–895) is devoted to "Bestimmung und Zusammenhang der astronomischen Konstanten" by J. Bauschinger.

In *Proceedings of the Benares Mathematical Society*, volume 2, 1920, D. K. Sen, research scholar in mathematics at Benares Hindu University, has an eleven page article entitled: "On the application of Burgess's method for determining the uniform motion of an ellipsoid of revolution through a viscous liquid along its axis of revolution." The reference here is to Professor R. W. Burgess's doctor's thesis at Cornell, on "The uniform motion of a sphere through a viscous liquid," published in the *American Journal of Mathematics*, 1916.

America's influence is being wielded in mathematics of the elementary and secondary schools of Cuba and South America through the works in Spanish by "Jorge Wentworth y David Eugenio Smith" published by the enterprising firm of Ginn and Company. These works are: *Geometría Plana y del de Espacio* (1915, 8 + 469 pp.; price $1.72; translation of the authors' English work), *Elementos de Algebra* (1917, 6 + 458 pp.; price $1.72; translation, with slight changes, of book 1 and part of book 2 of the authors' *School Algebra*), and *Aritmética Moderna*, 2 books (1916, 6 + 265 + 6 + 317 pp.; price $.64 + $.80; not an exact translation of any American editions).